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Hazard Analysis for Uncontrolled Space Vehicle Reentry

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Satellites in low Earth orbit ultimately reenter the Earth's atmosphere at the end of the mission due to orbital decay. Although most of the mass of a typical space vehicle is destroyed and rendered harmless, a significant portion survives to ground impact. As the number of reentry events has increased in recent years, some surviving components have impacted near populated areas and drawn attention to the casualty risk. In response to this hazard, the U.S. government developed guidelines to mitigate the danger posed by randomly reentering space objects that survive to surface impact. If an upper bound for casualty expectation is exceeded, a controlled deorbit/reentry into a sparsely populated region is recommended. This paper develops the mathematical methods needed to compute risk to people and property from uncontrolled reentries. The impact probability density function for surviving debris objects associated with an uncontrolled reentry is derived. The impact probability density function is used with the population density function to compute the casualty expectation. Examples of casualty expectation and risk of damage to property are provided.

Nomenclature

A = vehicle drag area
da = infinitesimal surface area
g = gravitation acceleration near Earth's surface
i = orbital inclination
L = latitude
M = debris mass
n = number of debris pieces per reentry event
P = probability of impact
p_i = probability of the ith debris piece producing a casualty
R = Earth's spherical radius
r_1 = effective radius of debris
r_2 = effective radius of a human
V = impact velocity
x = auxiliary parameter
β = ballistic coefficient
ΔA = impact area
δ = influence function
ε_i = casualty area
λ = latitude probability density function
$\rho(L)$ = population density as a function of latitude
ρ_o = atmospheric density at the Earth's surface
$\rho(x)$ = population probability density function
$\rho(\theta)$ = orbit probability density function
σ = impact probability density function
Ω = right ascension of the ascending node

Introduction

ORBITAL decay results in roughly 100 random reentries of large space objects per year, and this number is expected to increase in the future. Figure 1 illustrates the number and associated mass of reentries in recent years that involved larger space objects, not including service missions to the International Space Station or space shuttle activity.† The decrease in the number of reentries per year, shown in Fig. 1, is due in part to the decrease in solar activity associated with the 11-year solar cycle that peaked in approximately 2001. The number of reentries per year is expected to increase as we approach the next peak in the solar cycle, which is predicted to be near 2012.

Some recent reentry events involving recovered upper stages have demonstrated the danger of impacting debris [1–4]. Although the severe reentry heating environment consumes a significant fraction of the reentering mass, 10–40% of the prereentry mass is expected to survive to Earth surface impact [5].

Warnings are generally not given for randomly reentering space debris because impact location cannot be accurately predicted. Although all the space objects in Fig. 1 were tracked and had cataloged state vectors updated periodically, the time and location of each impact could not be accurately predicted due to unpredictable variations in high-altitude atmospheric density that alter the drag and orbital decay rate. Much of this variation is caused by unpredictable solar activity. As a result, the reentry time has an uncertainty of $\pm 10\%$ of the remaining orbit lifetime [6]. This uncertainty translates into uncertainty in impact location. For long-term prediction, the uncertainty is so large that it encompasses many orbital revolutions that span all values of longitude. Thus, any impact longitude is possible even for short-term predictions of about a day. For longer term prediction the impact longitude probability distribution is essentially uniform. Impact latitude distribution functions, however, are nonuniform and are bounded by $\pm i$, where i is the inclination of the parent orbit of the reentering object.

It is now clear that reentering space debris poses a hazard to people and property. How best to manage this risk has become an issue that is being addressed by the space community [7]. The U.S. government developed guidelines to limit the hazard from reentering space vehicles [8]. If the casualty expectation of a reentering satellite exceeds 1 in 10,000, a controlled deorbit is strongly recommended [8]. A controlled reentry implies that the object can be made to impact a desired location on the Earth's surface. Controlled reentries usually target a remote ocean area that is uninhabited. Thus, the risk associated with a successful controlled reentry is essentially zero. Controlled reentries were performed for both the Compton Gamma Ray Observatory (CGRO) [3,9] and the Mir Space Station. Controlled deorbit is also recommended for both the Hubble Space Telescope and the International Space Station (ISS) at the end of their respective missions. Less massive space vehicles are also subject to controlled deorbit, if the casualty expectation exceeds U.S. government guidelines. The recent controlled deorbit of the Delta IV Second Stage [10] is such an example.

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†Data available online at http://www.aero.org/capabilities/cords/reentry-stats.html [retrieved 1 August 2008].

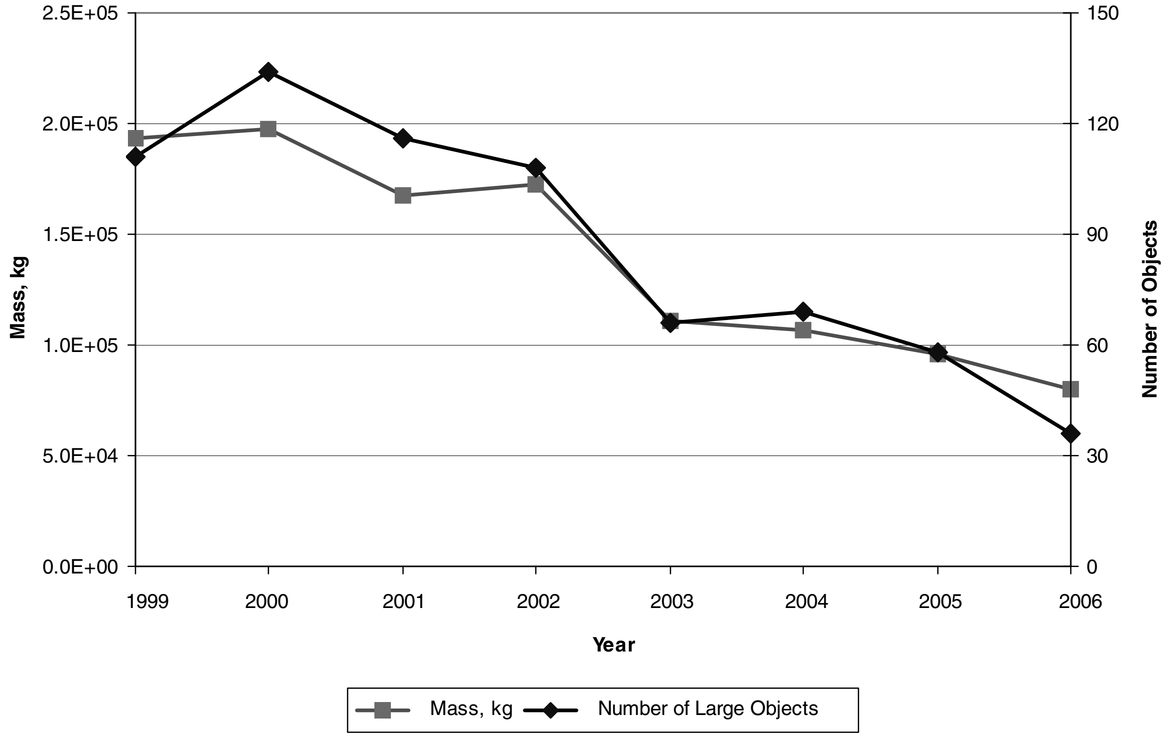

Fig. 1 Total mass and number of large object reentry for recent years.

Reentry risk can also be reduced with only partial control of a space vehicle via drag modulation [11]. This was achieved during the reentry of Skylab in 1979 by shifting the impact footprint thousands of miles away from North America [12]. Drag modulation has been proposed as a method of reducing reentry risk for larger space vehicles that would otherwise require expensive controlled deorbit systems [13].

Controlled reentries are performed only if absolutely necessary due to cost, mission impact, and difficulty in performing the maneuver. Therefore, it is important to calculate the casualty expectation to determine if a controlled deorbit is required.

Published methods to compute casualty expectation lack adequate mathematical foundations. For example, the derivations of the impact probability density function and casualty expectation for a debris object randomly reentering from an orbit of inclination, i, do not appear in the literature.

The goal of this work is to improve the mathematical methods needed to quantify the hazard due to uncontrolled space reentry. This is achieved by introducing a continuous function for impact probability density, which is expressed using simple algebraic and trigonometric functions. The impact probability density function is used to determine the risk to a building structure, ship, or aircraft, which depends on the location and associated area of the object at risk.

The impact probability density function is also used with the population density function to derive the integral expression for casualty expectation. This integral enables one to compute the casualty expectation per unit area for an object reentering from an orbit with arbitrary inclination. An interaction function is included in the formulation for mathematical rigor, as well as to quantify the severity of the debris impact, which can result in greater or fewer casualties depending on the impact energy. The casualty expectation integral can be easily evaluated and results in the casualty expectation per unit casualty area as a function of the orbital inclination of the reentering space object. The resulting plot [1] has been presented in a previous publication but the details of the underlying computations have never been published in an archival journal. The uncertainty in the casualty expectation is evaluated by considering all possible longitudes of the ascending node of the final orbit revolution. The results are applied to a Delta IV Upper Stage random reentry by calculating the casualty expectation distribution function.

This paper is organized into sections involving reentry survivability, impact probability density, global population density, and casualty expectation calculation. The reentry survivability section reviews information regarding reentry heating, vehicle breakup, and key factors in survivability. The section on impact probability density derives the analytic expression for the impact probability density function as a function of impact latitude and space vehicle orbital inclination. The section on global population density derives the population density as a function of impact latitude based on a gridded global population database. This is needed to complement the impact probability density, which also depends on latitude. The section on casualty expectation is the focus of the paper. Here the impact probability density is used with the global population density to derive the casualty expectation integral, which is used to calculate the casualty expectation per unit casualty area as a function of the orbital inclination. The variation in casualty risk as a function of the longitude of the ascending node of the final orbit revolution is also presented.

Reentry Survivability

The subject of reentry heating and space vehicle survivability is complex. The interested reader can consult numerous references on the subject [5,9,14–21]. Only a high-level qualitative review of the subject is presented here.

Over time, the orbits of space objects in low Earth orbit decay and tend to circularize due to the higher drag at perigee altitude. The enormous amount of orbital energy must be removed during the reentry process. Most of the energy goes into heating the atmosphere as the space object descends through the atmosphere. A fraction of the energy goes into heating the space object itself. Aerodynamically sleek shapes transfer more of the heat energy to the vehicle. In contrast, blunt-shaped objects transfer more of the heat to the atmosphere.

If the heating rate is sufficiently high, the space object skin or outer components can melt or vaporize. The heating rate depends on the speed at which the object descends through the atmosphere, which depends on the ballistic coefficient defined in Eq. (1):

$$\beta = mg/C_d A \tag{1}$$

where g is the gravitational acceleration at the Earth's surface, m is the mass of the debris piece, A is the drag area, and C_d is the drag coefficient. Although some analysts define the ballistic coefficient as $\beta = C_d A/M$, the definition in Eq. (1) is used in this work.

High ballistic coefficient objects descend at faster rates and have higher peak heating rates. Lower ballistic coefficient objects descend at lower rates through the atmosphere and have a longer amount of time to radiate the thermal energy away. By the time an object reaches the denser portions of the atmosphere, its velocity is lower; therefore, the peak heating is lower.

There are several processes that serve to reduce the predicted peak temperature during a reentry: heat capacity, ablation, and radiation. The heat capacity of the space object's skin serves, to some extent, as a heat sink to reduce the peak temperature. Because a heat sink absorbs heat throughout the reentry, its effectiveness is determined by the total heat load and not the peak heating rate. Thus, a heat sink is more effective against a rapid reentry that has a high peak heating rate but a lower total heat load.

Ablation is the vaporization of the space object's skin. Ablation can absorb a great deal of energy per pound of surface material. Thus, the heat energy that would go into raising the temperature goes into changing the phase of the skin material. In addition, the release of ablation products tends to impede the heat flow from the heated atmosphere to the object's skin.

Radiation serves to reduce the peak temperature by emitting the heat energy as electromagnetic radiation. Because the radiated power is proportional to the fourth power of temperature, it is more effective at high temperatures. The radiated power is proportional to the area of the emitting surface. Radiation helps reduce the total heat load. It is more effective for lower ballistic coefficient objects that are heated higher in the atmosphere, thus experiencing a lower heating rate spread over a longer time. In this manner, more heat is radiated away earlier in the descent and the velocity is reduced when the object enters the denser regions of the atmosphere.

Vehicle survivability prediction is complicated by vehicle breakup, which exposes shielded components to direct atmospheric heating. Thus, internal components of various ballistic coefficients are released at breakup and follow unique trajectories and heating profiles. Secondary breakups of released components are possible, and a cascade of secondary breakups can occur. Objects with the greatest likelihood of survival are lower ballistic coefficient pieces of blunt shape made of materials with a high melting temperature and high heat capacity. Spherical pressure vessels and propellant tanks have been recovered with relatively little physical damage caused by the reentry heating environment. Very large space vehicles tend to have more surviving components due to shielding and heat capacity effects. Skylab, Cosmos 954, and Salyut-7/Kosmos-1686 are examples of massive space vehicles that had a significant amount of surviving debris.

It should be noted that the on-orbit ballistic coefficient is typically significantly different from the reentry ballistic coefficient because the space vehicle breaks up due to aerodynamic and heating loads and because the value of C_d changes based on changes in the aerodynamic flow regime. Large solar panels usually break off first. As a result, there is a range of ballistic coefficients for space vehicle fragments after the main breakup event. An analysis of the Hubble Space Telescope indicated that 98 different components (some with multiple quantities) will survive the reentry environment. Variation in particulars, including ballistic coefficients, results in an impact footprint 1220 km long. The analysis estimated that 2055 kg of mass survives to impact, which represents 17.4% of the on-orbit mass [22]. An analysis of a Delta II Second Stage indicated that at least 331 kg (one propellant tank and four pressurization spheres) of the 920 kg Second Stage survived reentry [5]. That amounts to at least 36%. These two examples fall within the expected 10–40% of surviving mass stated earlier.

Space vehicles can be designed to reduce reentry survivability, and thereby reduce casualty risk. The easiest method is to choose low-melting-point materials for major components. It might be possible to construct the space vehicle in a manner that will assure a particular breakup scenario. For example, if the total vehicle is predicted to survive, it may be possible to design the vehicle to separate into two or more less-survivable segments during reentry. On the other hand, if more-survivable components tend to separate from the core vehicle early in the reentry, thereby avoiding peak heating, it might be possible to prevent this early separation. Keeping these more-survivable components attached to the core vehicle could subject them to higher heating loads sufficient to destroy these more resilient components. As more is learned about the reentry heating environment, more approaches to minimizing reentry survivability can be applied.

Although the amount of surviving debris can be reduced, it is likely that there will always be some debris objects that survive to surface impact. It is the surviving debris that impacts with sufficient energy to injure people or damage property that is the subject of this work.

Impact Probability Density

Objects that reenter the Earth's atmosphere via drag-induced decay tend to circularize before reentry. This is because drag at perigee lowers apogee and drag at apogee lowers perigee. Because drag at perigee is higher due to denser atmosphere, apogee decreases faster than perigee. Even if reentry occurs at a nonzero eccentricity, uncertainty in long-term prediction results in a uniform distribution of the reentry position within the orbit plane. That is, the angular position of the reentry point from the ascending node (argument of perigee plus true anomaly) is uniformly distributed. At low Earth orbit, the equatorial bulge causes the orbit plane to precess about the Earth's spin axis, thus changing the right ascension of the ascending node, Ω. The uncertainty in atmospheric drag over several days, which involve tens of orbits, results in a uniform distribution of the longitude of the ascending node, Ω. In addition, the uncertainty in reentry time randomizes the longitude range of the reentry footprint. Only for short-term predictions on the order of a few days is it possible to predict the longitude of the reentry footprint. Therefore, we assume that the impact longitude is uniformly distributed.

Let θ be the angle between the ascending node and the reentry position in the orbit plane. The probability density function for θ is given by

$$\rho(\theta) = 1/2\pi \tag{2}$$

The normalization condition is satisfied:

$$\int_0^{2\pi} \rho(\theta)\,\mathrm{d}\theta = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} = 1 \tag{3}$$

For each value of θ, a latitude angle can be computed. Here we neglect the slight oblate shape of the Earth and assume a spherical shape for simplicity. There are two values of θ for each value of latitude, one for ascending motion and the other for descending motion. The distribution of latitude for a random reentry is not uniform. The latitude distribution can be computed by relating θ to latitude via the intermediate Cartesian coordinate parameter z:

$$z = R\sin(L) = R\sin(\theta)\sin(i) \tag{4}$$

Eliminating R and taking the derivative of Eq. (4) yields

$$\cos(L)\,\mathrm{d}L = \cos(\theta)\sin(i)\,\mathrm{d}\theta = (\sin^2(i) - \sin^2(L))^{1/2}\,\mathrm{d}\theta \tag{5}$$

Thus, $\mathrm{d}\theta$ is related to $\mathrm{d}L$ by

$$\mathrm{d}\,\theta = \frac{\cos(L)\mathrm{d}L}{(\sin^2(i) - \sin^2(L))^{1/2}} \tag{6}$$

The impact location distribution function can be expressed in terms of latitude by considering an infinitesimal probability associated with $\mathrm{d}L$ and $\mathrm{d}\theta$:

$$\rho(\theta)\,\mathrm{d}\theta = \frac{\mathrm{d}\theta}{2\pi} = \lambda(L, i)\,\mathrm{d}L \tag{7}$$

Using Eq. (6) in Eq. (7) yields

$$\frac{\mathrm{d}\theta}{2\pi} = \lambda(L, i)\,\mathrm{d}L = \frac{\cos(L)\,\mathrm{d}L}{\pi(\sin^2(i) - \sin^2(L))^{1/2}} \tag{8}$$

where a factor of 2 was included to account for the fact that the range of L is π, whereas the range of θ is 2π. The latitude probability density function is therefore

$$\lambda(L, i) = \frac{\cos(L)}{\pi(\sin^2(i) - \sin^2(L))^{1/2}} \tag{9}$$

Notice that, if the inclination is $\pi/2$ radians, the latitude distribution becomes uniform as expected:

$$\lambda(L, (\pi/2)) = 1/\pi \tag{10}$$

The impact probability density on the Earth's surface is the probability per unit area. It can be obtained from Eq. (9) by simply dividing by the surface area associated with an infinitesimal latitude, dL:

$$\mathrm{d}\,a = 2\pi R^2 \cos(L)\,\mathrm{d}L \tag{11}$$

That is

$$\sigma(L, i) = \frac{\lambda(L, i)\,\mathrm{d}L}{\mathrm{d}a} = \frac{\lambda(L, i)\,\mathrm{d}L}{2\pi R^2 \cos(L)\,\mathrm{d}L} = \frac{1}{2\pi^2 R^2 (\sin^2(i) - \sin^2(L))^{1/2}} \tag{12}$$

The normalization condition is given by

$$\int_{-i}^{i} \sigma(L, i) 2\pi R^2 \cos(L)\,\mathrm{d}L = \int_{-i}^{i} \frac{\cos(L)\,\mathrm{d}L}{\pi(\sin^2(i) - \sin^2(L))^{1/2}} \tag{13}$$

Notice that the range of L is bounded by the orbital inclination. This integral can be evaluated by a change of variable given by

$$\sin(L) = \sin(i)\sin(x) \tag{14}$$

The derivate of Eq. (14) is

$$\cos(L)\mathrm{d}L = \sin(i)\cos(x)\,\mathrm{d}x \tag{15}$$

Using Eqs. (14) and (15) in Eq. (13) gives the proper normalization as expected:

$$\int_{-\pi/2}^{\pi/2} \frac{\sin(i)\cos(x)\,\mathrm{d}x}{\pi(\sin^2(i) - \sin^2(i)\sin^2(x))^{1/2}} = \int_{-\pi/2}^{\pi/2} \frac{\mathrm{d}x}{\pi} = \frac{1}{\pi}\left(\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right) = 1 \tag{16}$$

One can use Eq. (12) to compute the probability that a piece of reentering debris will strike an area on the Earth's surface. If a building is located at latitude 30 deg north or south and has an area of 1000 ft^2 and a piece of debris reenters the Earth's atmosphere from an orbit having an inclination of 45 deg, the impact probability is given by

$$P = \Delta A \sigma(L, i) = 1000\sigma((\pi/6), (\pi/4)) = 1000(2.314 \times 10^{-16}) = 2.314 \times 10^{-13} \tag{d17}$$

Notice that the longitude of the building is not important for this random reentry. This result indicates that the probability of any one single building being struck by space debris is quite small.

One can compute the probability of an unsheltered person being struck by a piece of space debris in a similar fashion. If the area of an average person is 4 ft^2, then the probability of impact for 30 deg latitude is

$$P = \Delta A \sigma(L, i) = 4\sigma((\pi/6), (\pi/4)) = 4(2.314 \times 10^{-16}) = 9.255 \times 10^{-16} \tag{18}$$

If the entire global population of about 6 billion people was located at latitude 30 deg north or south and distributed in longitude to prevent occupying the same location, the probability of a single person being struck by a debris piece is given by

$$P = 6 \times 10^9 \Delta A \sigma(L, i) = 24 \times 10^9 \sigma((\pi/6), (\pi/4)) = 24 \times 10^9 (2.314 \times 10^{-16}) = 5.553 \times 10^{-6} \tag{19}$$

This calculation assumes that the population is unsheltered and neglects the area of the debris piece itself. It is meant to simply illustrate the use of the impact probability density in Eq. (12). To compute the casualty expectation associated with reentering debris, one must properly treat the global population density. In addition, the interaction between the debris and population must be included in the formulation.

Global Population Density

Unlike impact probability density, the global population density cannot be represented by a simple analytic function. Instead, the population density can be computed numerically using a database containing population as a function of latitude and longitude [23,24]. The longitude dependence only matters if the location of the reentry impact footprint is fairly well known. Because we are only interested in random reentry for which the location of impact is unknown, we assume the longitude of the impact footprint is uniformly distributed. Therefore, we only need the population density as a function of latitude for casualty expectation analysis.

The most recent available global population database was processed to obtain the population density as a function of latitude [23]. Figure 2 illustrates the population distribution function as a function of latitude. Figure 3 contains the population density as a function of the sine of the latitude, which is needed later for computing casualty expectation. Also illustrated is the distribution function, which is essentially the integral of the probability density starting in the southern hemisphere and integrated northward through the northern hemisphere. One can account for population density in years later than 1995 by simply assuming a constant 1% growth rate. If the growth rate is asymmetric, with the southern hemisphere having a 2% per year growth rate and the northern hemisphere having a 0.5% per year growth rate, the distribution does not change significantly when extrapolated to the year 2007, as illustrated in Figs. 2 and 3. The higher population density in the northern hemisphere is clearly visible in both Figs. 2 and 3.

Although the population density as a function of latitude cannot be expressed using elementary functions, it can be computed and tabulated for related computations, such as casualty expectation.

Casualty Expectation Calculation

The casualty expectation calculation requires that the reentering debris impact probability density be coupled to the population density function. Impacting debris can only affect population in the vicinity of the impact point. The closer the debris impact point is to a person, the greater the chance of casualty. This coupling between the debris impact point and population density function can be represented mathematically by an influence function, $\delta(x)$, where x is the separation distance between a small area with a population density and debris impact point. The influence function relates the magnitude of the casualty to the separation distance between a person and a debris object. The influence function includes the effective area of the debris piece, the average area of a human, and the energy of the impacting debris, as well as a structure that may protect the human population. In this analysis, we do not require the detailed form of the influence function, but just its integral over the region of interest. This is because both the impact probability density and population

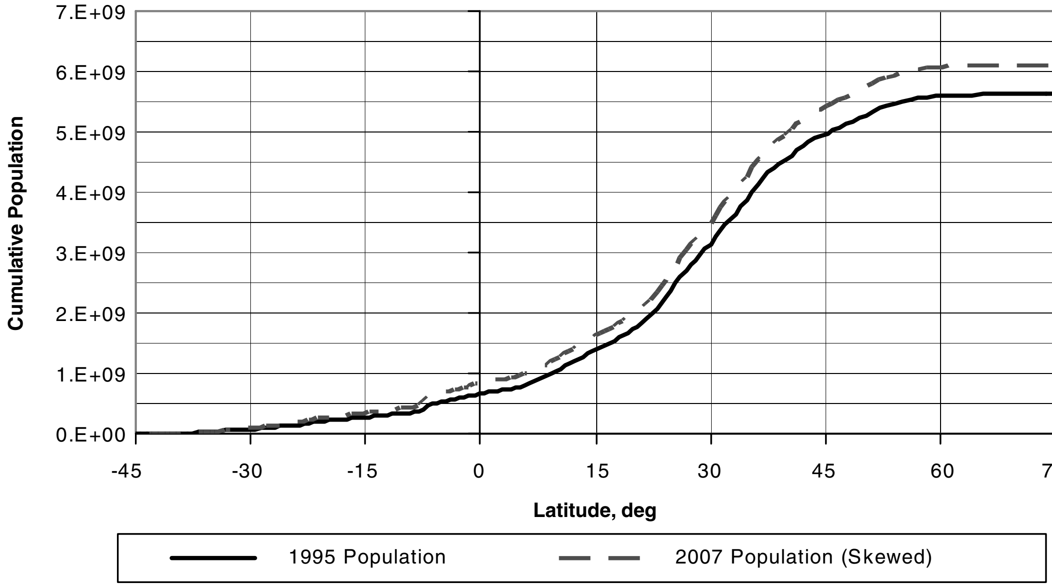

Fig. 2 Population distribution function.

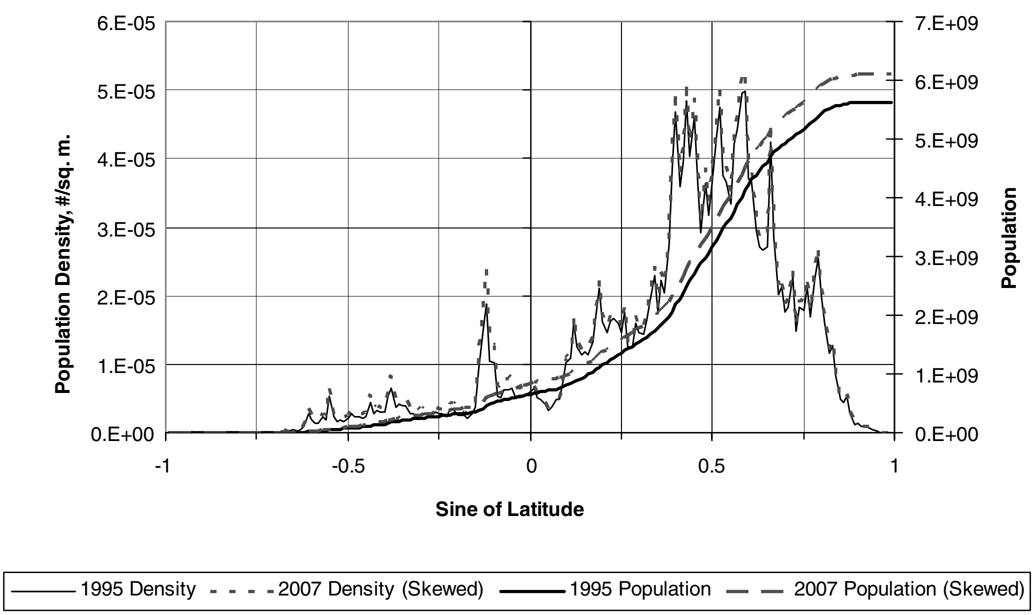

Fig. 3 Population density functions.

density vary very little over the impact region. Thus, we assume a step function centered on the debris impact point.

The casualty expectation depends on the probability density of a debris object, $\sigma(x)$; the probability density of the population, $\rho(x)$; and the influence function, $\delta(x)$. The probability that a debris object impacts the infinitesimal area, $\mathrm{d}a'$, is given by $\sigma(x')\mathrm{d}a'$. The number of people residing in the infinitesimal area, $\mathrm{d}a$, is given by $\rho(x)\mathrm{d}a'$. The amount that the impacting debris at position x' will affect the population at x is given by $\delta(x - x')$. The casualty expectation is computed as the product of these functions integrated over the respective areas. Expressed mathematically, one finds for the ith debris object

$$p_i = \iint_{a,a'} \rho(\mathbf{x})\sigma(\mathbf{x}')\delta(\mathbf{x} - \mathbf{x}')\,\mathrm{d}a\,\mathrm{d}a' \tag{20}$$

One can perform the $\mathrm{d}a$ integration first. Because the impact probability density does not vary much over the integration region, a', $\sigma(x')$ equals $\sigma(x)$ and it can be brought outside of the integral:

$$\varepsilon_i\sigma(\mathbf{x}) = \sigma(\mathbf{x})\int_{a'} \delta(\mathbf{x} - \mathbf{x}')\,\mathrm{d}a' = \int_{a'} \sigma(\mathbf{x}')\delta(\mathbf{x} - \mathbf{x}')\,\mathrm{d}a' \tag{21}$$

In Eq. (21), ε_i is an effective interaction area centered on x and is given by

$$\varepsilon_i = \int_{a'} \delta(\mathbf{x} - \mathbf{x}')\,\mathrm{d}a' \tag{22}$$

The functional form of $\delta(x)$ is not important, only its integral over region a'. Therefore, one can assume a step function for simplicity and interpret ε_i as the effective casualty area for the ith debris object.

The parameter ε_i is also used to quantify the severity of the influence of the debris impact. If the size, mass, and energy of the impacting debris are large, then ε_i will be large as well. On the other hand, if the population is sheltered by strong housing units, ε_i will be smaller. A 100% sheltering of the population implies that ε_i is 0. Using Eq. (21) in Eq. (20) yields

$$p_i = \varepsilon_i \int_a \rho(\mathbf{x})\sigma(\mathbf{x})\,\mathrm{d}a \tag{23}$$

The problem is now reduced to calculating the densities, ρ and σ, appropriate for a given problem and evaluating the integral. Equation (23) is valid for both random and controlled reentry. This paper is concerned with random reentry, and so the impact probability density is a function of latitude and orbital inclination only, as derived earlier. The global population density as a function of latitude can be computed from a population database and is illustrated in Fig. 3.

Using Eqs. (11) and (12) in Eq. (23), we find

$$p_i = \frac{\varepsilon_i}{\pi}\int_{-i}^{i} \frac{\rho(\sin(L))\cos(L)}{((\sin^2(i)) - \sin^2(L))^{1/2}}\,\mathrm{d}L \tag{24}$$

The integral can be broken into its southern and northern hemisphere contributions.

$$p_i = \frac{\varepsilon_i}{\pi}\int_{-i}^{0} \frac{\rho(\sin(L))\cos(L)\,\mathrm{d}L}{(\sin^2(i) - \sin^2(L))^{1/2}} + \frac{\varepsilon_i}{\pi}\int_{0}^{i} \frac{\rho(\sin(L))\cos(L)\,\mathrm{d}L}{(\sin^2(i) - \sin^2(L))^{1/2}} \tag{25}$$

It is convenient to change the integration variable from L to $-L$ in the southern hemisphere integral, so that Eq. (25) becomes

$$p_i = \frac{\varepsilon_i}{\pi}\int_{0}^{i} \frac{\rho(\sin(-L))\cos(L)\,\mathrm{d}L}{(\sin^2(i) - \sin^2(L))^{1/2}} + \frac{\varepsilon_i}{\pi}\int_{0}^{i} \frac{\rho(\sin(L))\cos(L)\,\mathrm{d}L}{(\sin^2(i) - \sin^2(L))^{1/2}} \tag{26}$$

Changing the integration variable from L to i by noting that $\sin(L) = \sin(i)\sin(\theta)$, one finds

$$p_i = \frac{\varepsilon_i}{\pi}\int_{0}^{\pi/2} \rho(-\sin(i)\sin(\theta))\,\mathrm{d}\theta + \frac{\varepsilon_i}{\pi}\int_{0}^{\pi/2} \rho(\sin(i)\sin(\theta))\,\mathrm{d}\theta \tag{27}$$

Because the integrands are simply the population density as a function of the sine of latitude, as illustrated in Fig. 3, these integrals are easily evaluated. Figure 4 illustrates p_i as a function of inclination for the sum of populations in both the northern and southern hemispheres. Figure 5 shows the contribution of each hemisphere to the casualty expectation.

The peak in Fig. 4 occurs near a 35 deg inclination and indicates a high population density in the northern hemisphere near 35 deg latitude. The contribution from the southern hemisphere, is small as illustrated in Fig. 5. The large peak associated with the southern hemisphere in Fig. 5 is due to the large population in Indonesia.

It may be possible to predict the longitude of the ascending node of the final orbital revolution closer to the time of reentry. For a given inclination and longitude of the ascending node of the final orbital revolution, the ground track can be computed. Using the population near the ground track, one can compute the casualty risk relative to that of the random reentry casualty risk [13]. It can be greater or less than the random risk by a factor termed the "fractional risk." The average of all fractional risk values associated with all ascending node values is 1. Figure 6 illustrates the fractional risk as a function of the longitude of the ascending node for the final orbital revolution of an orbit inclined at 10 deg. Variation in the fractional risk is due to the nonuniform distribution of the Earth's population in both latitude and longitude. If the longitude of the ascending node is -150 deg, the fractional risk is 75% of that obtained from Fig. 4. The casualty expectation in this case is $0.75 \times 8.5 \times 10^{-6} = 6.4 \times 10^{-6}$. If it is known with certainty that the reentry will be in the southern hemisphere, one can use Fig. 5 to obtain the casualty expectation for an orbit inclined at 10 deg. In this case, the casualty expectation becomes $0.75 \times 3.3 \times 10^{-6} = 2.5 \times 10^{-6}$. If the longitude of the ascending node is -105 deg, the fractional risk is about 3 times larger than that of Fig. 4. Thus, the casualty risk for an ascending node of -105 deg is 4 times greater than that of a -150 deg ascending node. If it is possible to change the orbital decay rate to change the longitude of the ascending node, the casualty risk can be reduced.

Figures 6–8 illustrate the fractional risk vs longitude of the ascending node for orbital inclinations of 28.5, 35, 51, 82, and 98 deg, respectively. The figures indicate that the fractional risk can vary by an order of magnitude, whereas the average fractional risk is equal to 1 in each case. The values of fractional risk can be used with the values from Fig. 4 to compute the casualty expectation per square meter of casualty area.

The damage caused by impacting debris can vary depending on the size, mass, and drag coefficient. Atmospheric drag slows most debris objects to their free fall state before surface impact. A

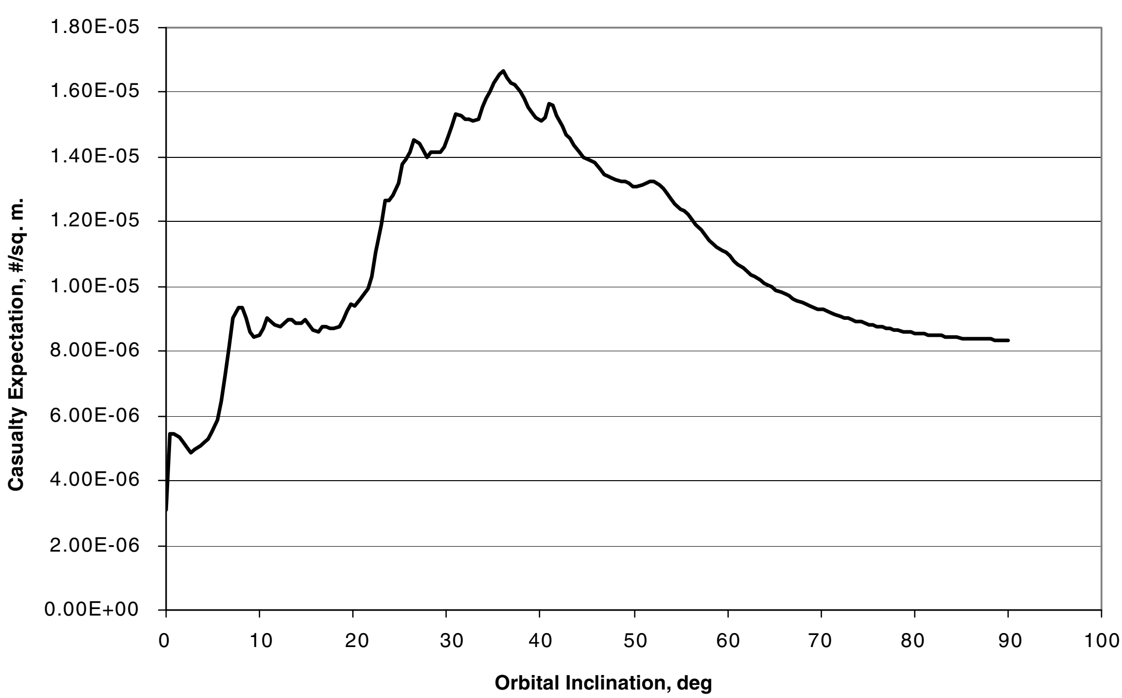

Fig. 4 Casualty expectation for a random reentry as a function of orbital inclination.

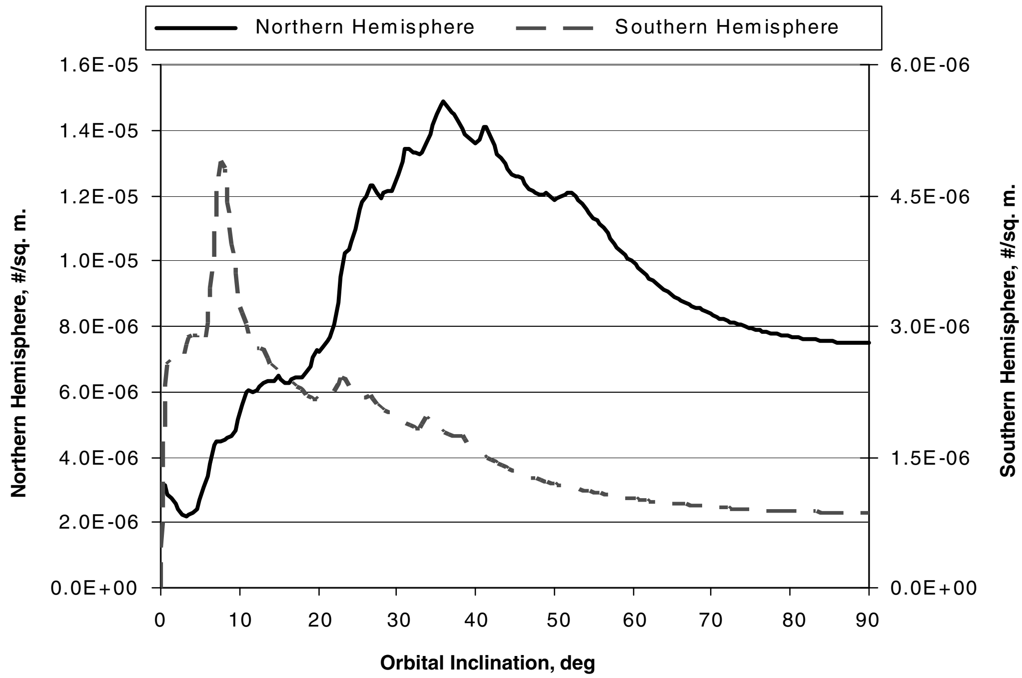

Fig. 5 Casualty expectation for a random reentry for both the northern and southern hemispheres.

computer simulation was used to illustrate the situation at impact by placing objects with a range of ballistic coefficients in an 80 n mile circular orbit and allowing them to decay to Earth impact. Figure 9 illustrates the acceleration and flight-path angle at impact, when free fall is equivalent to a vertical acceleration of 0 g. Because most space vehicle components that survive reentry have ballistic coefficients of less than 200, they impact nearly vertically at essentially their respective terminal velocities. A component with a very high ballistic coefficient of 500 would experience an acceleration of less than 20% of 1 g at impact.

Because some pieces of debris are not very massive and may have a low impact velocity, they may not have sufficient energy to pose a hazard. Therefore, debris pieces should be screened based on impact energy before they are included in the casualty expectation (CE) calculation. Hazards to people from falling debris have been studied in depth [25]. To simplify modeling, one can exclude objects with impact energies of less than a chosen threshold in the range of 15–35 ft · lbs[16]. The impact velocity and energy can be expressed in terms of object mass and ballistic coefficient:

$$V_I = \sqrt{\frac{2\beta}{\rho_o}} \tag{28}$$

$$E_I = m\beta/\rho_o \tag{29}$$

where m is the mass of the debris piece and r_o is the atmospheric density at the Earth's surface. As an example, a 1 lb object with a ballistic coefficient of 12 $\mathrm{lbs/ft^2}$ impacts the ground at 104 ft/s with 167 ft · lbs of energy. From Fig. 9, it is clear that an object with a

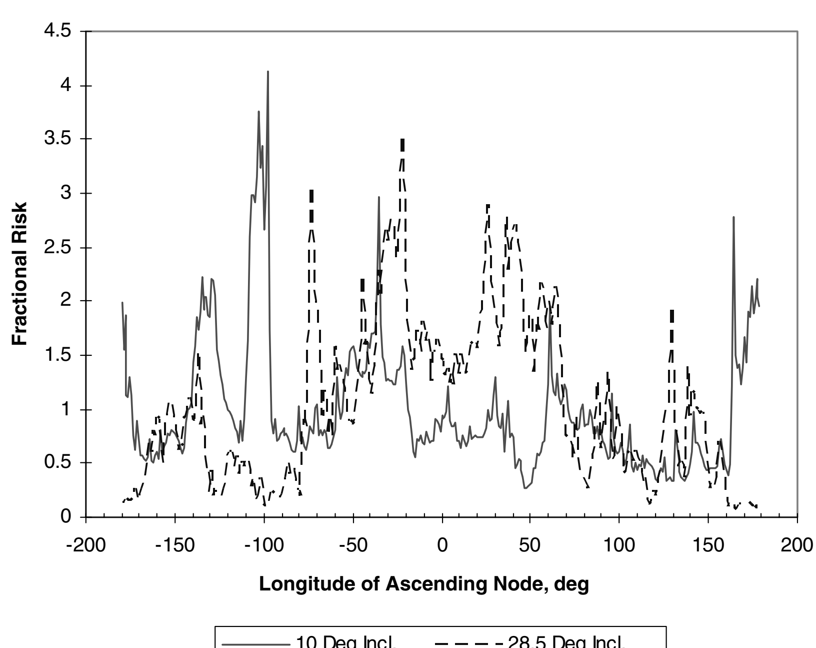

Fig. 6 Fractional risk as a function of longitude of ascending node for orbital inclinations of 10 and 28.5 deg.

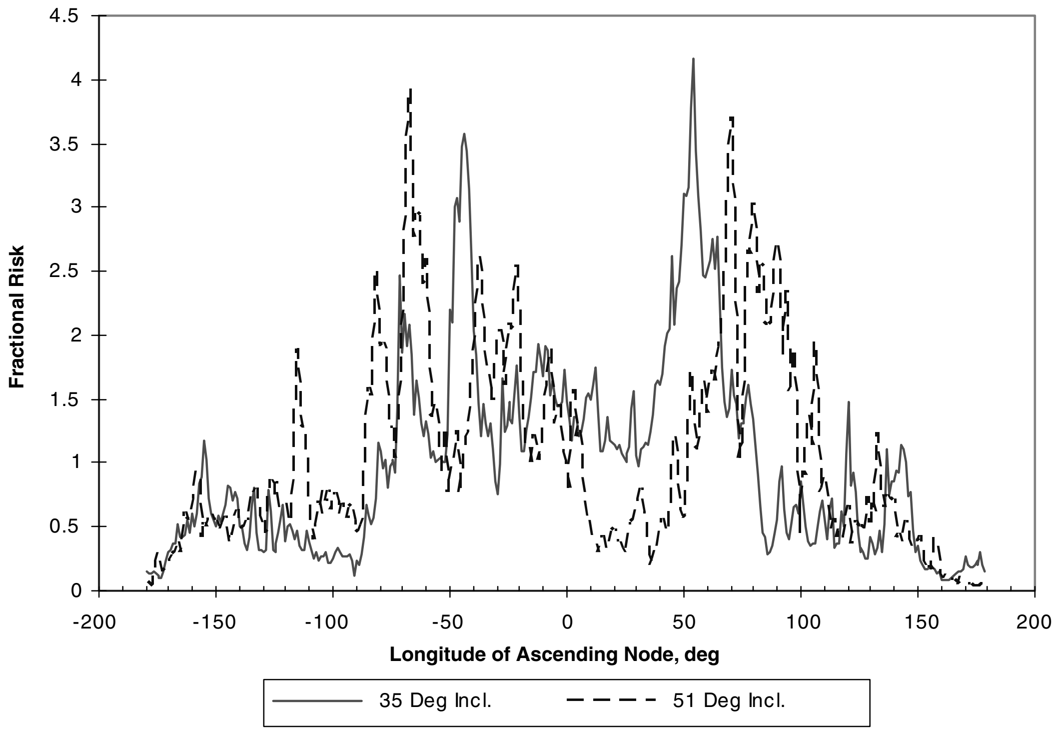

Fig. 7 Fractional risk as a function of longitude of ascending node for orbital inclinations of 35 and 51 deg.

ballistic coefficient of 12 $\mathrm{lbs/ft^2}$ is in a state of free fall with a -90 deg flight path, which corresponds to vertical descent.

The effective area used for each piece of debris in the CE calculation must be increased due to the finite size of a typical person. Each dimension should be increased by a man border in the range of 0.5–1.0 ft. For example, a piece of debris with an impact cross-sectional area of 3 ft by 2 ft is increased to 4 ft by 3 ft when given a 0.5 ft man border. A debris object with a circular cross-sectional radius of 2 ft is increased to 2.5 ft when given a 0.5 ft man border.

If the debris is approximately circular, let its radius by represented by r_1. Let the average area of a human be represented by radius r_2. For a debris object of area A, one finds its effective radius:

$$r_1 = \sqrt{A/\pi} \tag{30}$$

Thus, the effective interaction area for debris piece and human is given by

$$\varepsilon_i = (r_1 + r_2)^2 \pi \tag{31}$$

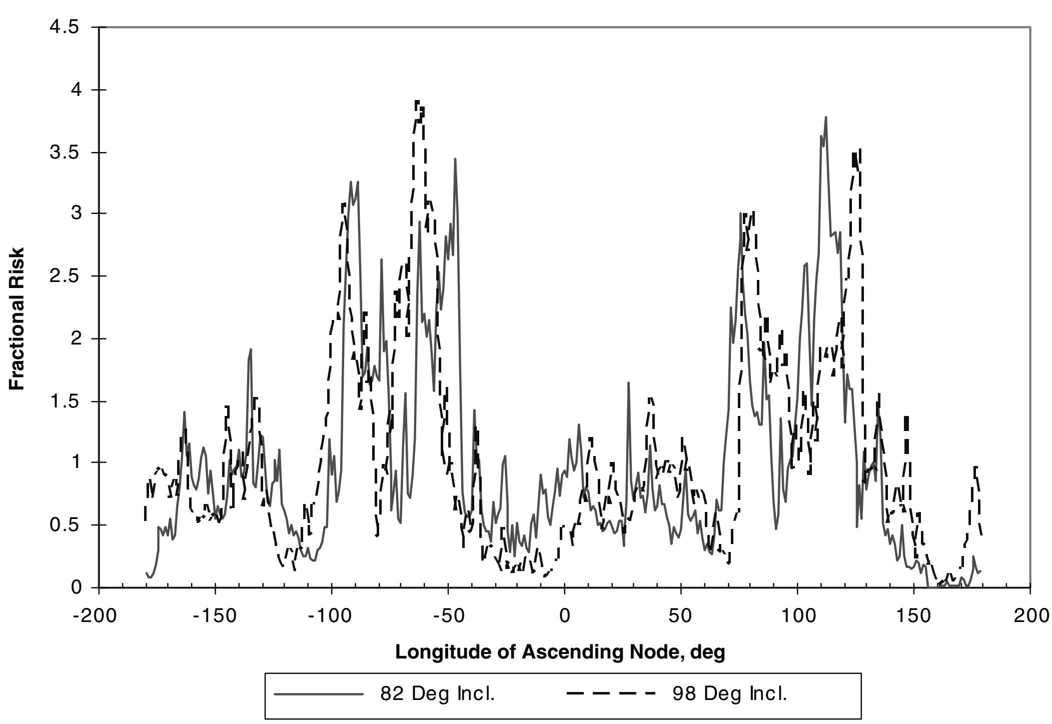

Fig. 8 Fractional risk as a function of longitude of ascending node for orbital inclinations of 82 and 98 deg.

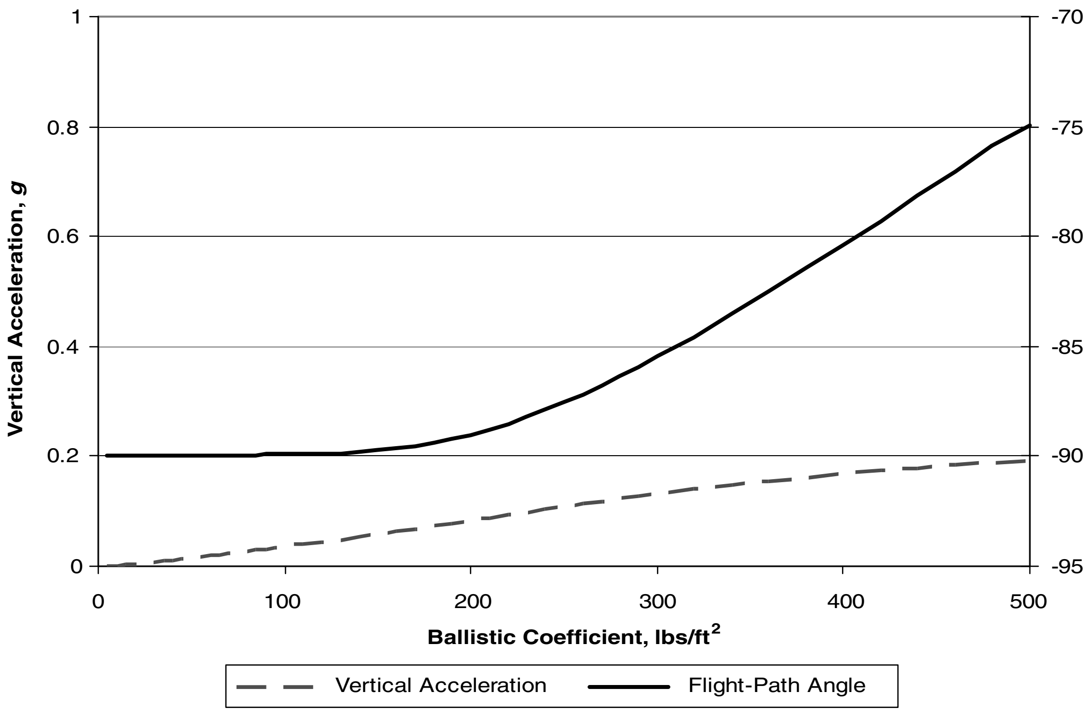

Fig. 9 Vertical acceleration and flight-path angle at impact for various values of ballistic coefficient.

If the debris object is shaped more like a rectangle than a circle, the effective interaction area should be computed by increasing its length and width appropriately:

$$\varepsilon_i = (l + 2r_2)(w + 2r_2) \tag{32}$$

The effective interaction area is computed for each debris object and used in Eq. (27) to calculate the respective casualty expectations. The total casualty expectation is obtained by summing over all n debris objects:

$$\mathrm{CE} = \sum_{i=1}^{n} p_i \tag{33}$$

Because the population is expected to continue growing at 1.099% per year, the casualty expectation will increase at the same rate. Because the population data in Fig. 3 is from 1995, the casualty expectation in years beyond 1995 is obtained from

$$\mathrm{CE}\,(t) = \mathrm{CE}(1995)(1 + 0.01099)^{(t-1995)} \tag{34}$$

Equation (34) should be updated when a revised global population database becomes available.

An example of the use of the casualty expectation calculation is provided in the decision to deorbit a Delta IV Medium Upper Stage [10] on 4 November 2006. The Upper Stage was used to place DMSP-17 into its sun synchronous orbit inclined at 98.7 deg. A reentry survivability analysis for the Upper Stage determined that the

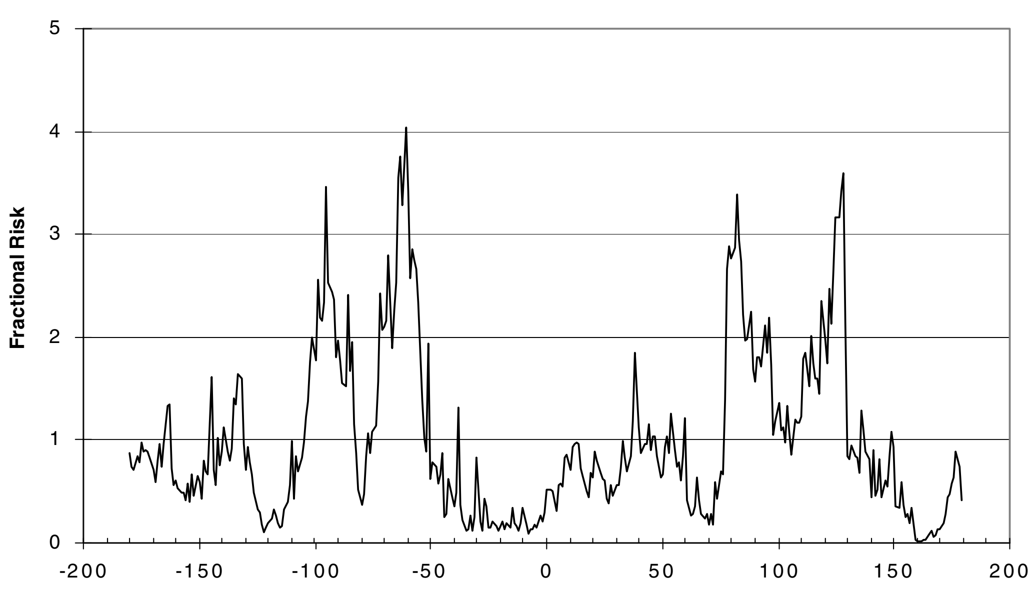

Fig. 10 Fractional variation in casualty expectation as a function of longitude of ascending node of the final orbit revolution for the Delta IV Upper Stage, which has an orbital inclination of 98.7 deg.

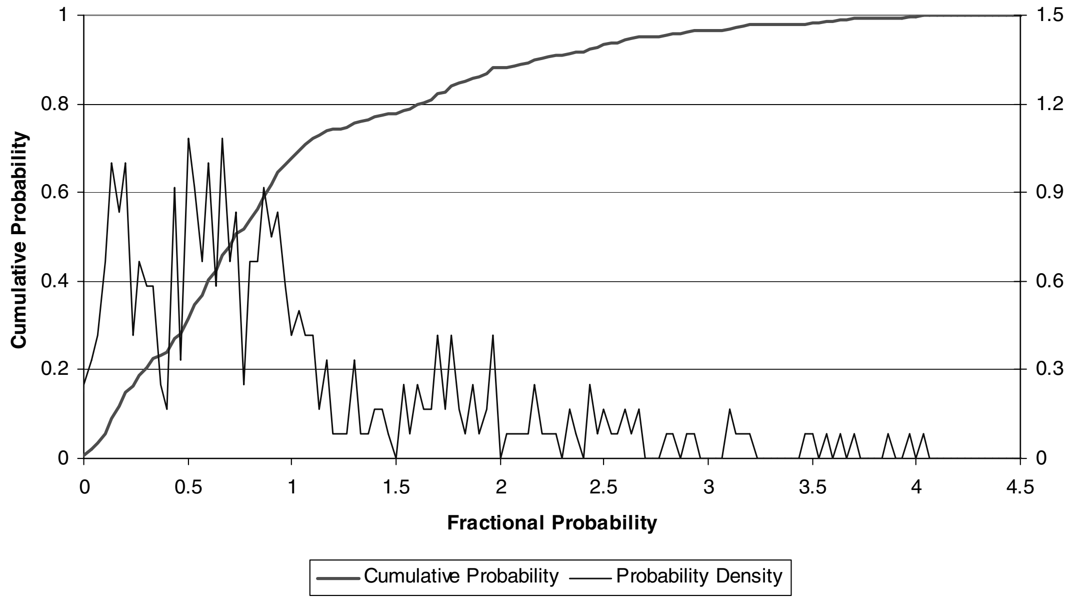

Fig. 11 Statistical variation of fractional risk illustrated in Fig. 10.

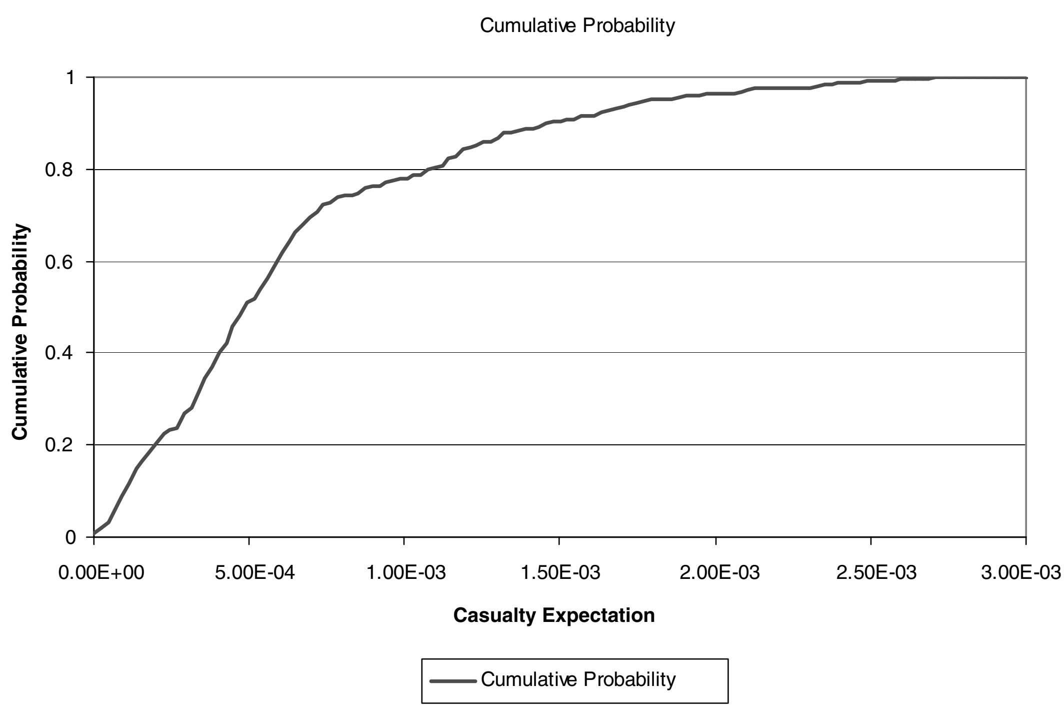

Fig. 12 Cumulative distribution function for the casualty expectation for the Delta IV Upper Stage.

casualty area of surviving debris would be about 70 m^2. Initial plans involved a perigee lowering maneuver that would have resulted in a random reentry. The casualty expectation per unit area for a random reentry was obtained from Fig. 4 by noting that risk from an orbit inclined at 98.7 deg is equivalent to an orbit inclined at 81.3 deg due to symmetry considerations. From Fig. 4, 8.51×10^{-6} casualties per square meter are expected based on 1995 population data. This results in 5.96×10^{-4} for the expected 70 m^2 of casualty area. Updating this number based on population growth expressed in Eq. (34) results in a casualty expectation of 6.72×10^{-4} casualties.

The uncertainty in the casualty expectation can be obtained by analyzing the fractional risk data for an orbit inclined at 98.7 deg, as shown in Fig. 10. The probability distribution and cumulative probability for the fractional risk is illustrated in Fig. 11. It indicates that there is a 20% chance of the fractional probability being above 1.6, which corresponds to a casualty expectation of 1.08×10^{-3}. The cumulative distribution for the total casualty expectation is shown in Fig. 12. It indicates that there is a 50% chance of the casualty expectation being above 4.93×10^{-4}. Because both of these numbers are significantly greater than the 1×10^{-4} guideline, a decision was made to perform a controlled deorbit. It should be noted that this deorbit was only possible because of the significant performance reserve on the Upper Stage. Not all upper stages have sufficient performance reserve to achieve deorbit.

Conclusions

Some debris from objects that reenter the Earth's atmosphere can survive the reentry heating environment and impact the Earth's surface. This impacting debris can injure people and damage property. An analytical expression for the impact probability density of a surviving debris object can be used to quantify the risk to a structure on the Earth's surface. The impact probability density was found to depend on the inclination of the space vehicle's orbit and the latitude of impact. Impact is not possible for latitudes with greater magnitudes than the orbital inclination. The highest impact probability density is at a latitude equal to the inclination of the space object orbit. The impact probability density is a symmetric function of latitude; therefore, 20 deg north latitude has the same impact probability density as 20 deg south latitude. Orbits with smaller inclinations have less potential area to impact. As a result, smaller inclination orbits have higher associated impact probability densities.

The impact probability density multiplied by the exposed area of a property in question equals the probability of impacting the property. The probability of impacting a particular building was found to be very small due to the large surface area of the Earth. The damage sustained at impact depends on the kinetic energy of the debris.

The casualty expectation associated with a space vehicle undergoing a random reentry is needed to determine if a controlled deorbit is necessary to keep the risk below U.S. government recommended guidelines. The population density can be used with the impact probability density to determine the casualty expectation per unit area of surviving debris. Although the population density has a longitudinal dependence, only the latitude dependence is significant due to the uniform longitude distribution of the impact probability density of the debris object. A global population database was used with the impact probability density to compute the casualty expectation per unit area as a function of the orbital inclination of the reentering debris. One needs only the casualty area of the surviving debris pieces and the precomputed casualty expectation per unit area as a function of the orbital inclination to compute casualty expectation. Although methods to reduce reentry survivability are being developed for future space vehicles, the need to perform casualty expectation calculations will remain for years to come.

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